Interview with John G. Thompson and Jacques Tits

Martin Raussen and Christian Skau

John G. Thompson and Jacques Tits are the recipients of the 2008 Abel Prize of the Norwegian Academy of Science and Letters. On May 19, 2008, prior to the Abel Prize celebration in Oslo, Thompson and Tits were jointly interviewed by Martin Raussen of Aalborg University and Christian Skau of the Norwegian University of Science and Technology. This interview originally appeared in the September 2008 issue of the *Newsletter of the European Mathematical Society*.

Early Experiences

Raussen & Skau: *On behalf of the Norwegian, Danish, and European Mathematical Societies we want to congratulate you for having been selected as Abel Prize winners for 2008. In our first question we would like to ask you when you first got interested in mathematics: Were there any mathematical results or theorems that made a special impression on you in your childhood or early youth? Did you make any mathematical discoveries during that time that you still remember?*

Tits: I learned the rudiments of arithmetic very early; I was able to count as a small child, less than four years, I believe. At the age of thirteen, I was reading mathematical books that I found in my father's library and shortly after, I started tutoring youngsters five years older than me who were preparing for the entrance examination at the École Polytechnique in Brussels. That is my first recollection. At that time I was interested in analysis but later on, I became a geometer. Concerning my work in those early years, I certainly cannot talk about great discoveries, but I think that some of the results I obtained then are not without interest.

My starting subject in mathematical research has been the study of strictly triple transitive groups; that was the subject essentially given to me by my professor [Paul Libois]. The problem was this: We knew axiomatic projective geometry in dimension greater than one. For the one-dimensional case, nobody had given an axiomatic definition. The one-dimensional case corresponds to $PSL(2)$. My teacher gave me the problem of formulating axiomatics for these groups. The idea was to take triple transitivity as the first axiom. So I started by this kind of problem: giving axiomatics of projective geometry based on triple transitivity. Of course, I was then led naturally to consider quadruple and quintuple transitivity. That is how I rediscovered all the Mathieu groups, except, strangely enough, the biggest one, the quintuple transitive. I had to rediscover that one in the literature!

R & S: *So you didn't know about the Mathieu groups when you did this work?*

Tits: No, I didn't.

R & S: *How old were you at that time?*

Tits: Eighteen years old, I suppose. In fact, I first found all strictly quadruple transitive groups. They were actually known by Camille Jordan. But I didn't know the work of Camille Jordan at the time. I rediscovered that.

R & S: *You must have been much younger than your fellow students at the time. Was it a problem to adjust in an environment where you were the youngest by far?*

Tits: I am very grateful to my fellow students and also to my family, because I was what is sometimes called a little genius. I was much quicker than all the others. But nobody picked up on that, they just let it go. My father was a little bit afraid that I would go too fast. My mother knew that this was exceptional, but she never boasted about it. In fact, a female neighbor said to my mother: "If I had a son like that, I would go around and boast about it." My mother found that silly. I was not at all put on a pedestal.

R & S: *Hardy once said that mathematics is a young man's game. Do you agree?*

Tits: I think that it is true to a certain extent. But there are people who do very deep things at a later age. After all, Chevalley's most important work was done when he was more than forty years old and even perhaps later. It is not an absolute rule. People like to state such rules. I don't like them really.

Thompson: Well, it is true that you don't have any childhood geniuses in politics. But in chess, music, and mathematics, there is room for childhood exceptional-

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ism to come forth. This is certainly very obvious in the case of music and chess and to some extent in mathematics. That might sort of skew the books in a certain direction.

As far as Hardy's remark is concerned I don't know what he was feeling about himself at the time he made that remark. It could be a way for person to say: "I am checking out now, I reached the age where I don't want to carry on." I don't know what the sociologists and psychologists say; I leave it to them. I have seen mathematicians reach the age of fifty and still be incredible lively. I don't see it as a hard and fast rule. But then Tits and I are really in no position to talk given our age.

R & S: *John von Neumann said, exaggerating a little, that whatever you do in mathematics beyond thirty is not worth anything, at least not compared to what you had done before thirty. But when he himself reached the age of thirty, he pushed this limit. Experience comes in, etc.*

Thompson: But he was a prodigy. So he knows the childhood side of it.

Tits: We all have known very young and bright mathematicians. The point is that to find deep mathematics, it is not necessary to have all the techniques. They can find results that are deep without having all of those techniques at hand.

R & S: *What about your memories on early mathematical experiences, Professor Thompson?*

Thompson: I don't have any particularly strong memories. I have an older brother, three years older than me, who was very good at math. He was instrumental, I guess, in interesting me in very elementary things. He was obviously more advanced than I was.

We also played cards in our family. I liked the combinatorics in card play. At that time, I was ten or twelve years old. I also liked playing chess. I never got any good at it but I liked it. When my brother went to the university, he learned about calculus and he tried to explain it to me. I found it totally incomprehensible, but it intrigued me. I did get books out of the library myself. But I didn't make too much progress without him.

Early Group Theory

R & S: *You have received this year's Abel Prize for your achievements in group theory. Can we start with a short historical introduction to the subject? We would like to ask you to tell us how the notion of a group came up and how it was developed during the nineteenth century. In fact, Norwegian mathematicians played quite an important role in that game, didn't they?*

Tits: Well, when you talk about groups it is natural to talk about Galois. I think Abel did not use groups in his theory—do you know?

Thompson: At least implicitly. I think the equation of the fifth degree comes in there. It was a great eye opener. I myself looked at a very well-known paper of Lagrange, I think around 1770, before the French revolution. He examined equations and he also said something about equations of degree five. He was definitely getting close to the notion of a group. I don't know about the actual formal definition. I guess we have to attribute it to Galois. Anyway, it was certainly he who came up with the notion of a normal subgroup. I am pretty sure that was Galois's idea. He came up with the idea of a normal subgroup, which is really basic.

Tits: But the theorem on the equation of degree five was discovered first by Abel, I think. Of course Galois had a technique that helped with many equations of different types that Abel did not have. Galois was really basically an algebraist, whereas Abel was also an analyst. When we now talk about abelian functions, these ideas go back to Abel.

R & S: *Can you explain why simple groups are so important for the classification of finite groups in general? That realization came about, we guess, with Camille Jordan and his decomposition theorem. Is that correct?*

Tits: You see, I think that one of the dreams of these people was always to describe *all* groups. And if you want to describe all groups you decompose them. The factors are then simple. I think that was one of the aims of what they were doing. But of course they didn't go that far. It is only very recently that one could find all finite simple groups, a solution to the problem to which John Thompson contributed in a major way.

R & S: *What about Sylow and Lie in the beginning of group theory?*

Thompson: Those are two other Norwegians.

Tits: Lie played an important role in my career. In fact, practically from the beginning, the main subject of my work has centered around the so-called exceptional Lie groups. So the work of Lie is basic in what I have done.

R & S: *Could you comment on the work of Frobenius and Burnside?*

Thompson: Of course. After the last half of the nineteenth century Frobenius among other things put the theory of group characters on a solid basis. He proved the orthogonality relations and talked about the transfer map. Burnside eventually got on the wagon there. And eventually he proved his $p^a q^b$- theorem, the two prime theorem, using character theory, namely that groups of these orders are solvable. That was a very nice step forward, I feel. It showed the power of character theory, which Frobenius had already done. Frobenius also studied the character theory of the symmetric groups and multiply transitive permutation groups. I don't know how much he thought of the Mathieu groups. But they were pretty curious objects that had been discovered before character theory. For a while there was quite a bit of interest in multiply transitive permutation groups, quite complicated combinatorial arguments. Burnside and Frobenius were very much in the thick of things at that stage.

Tits: When I was a young mathematician. I was very ignorant of the literature. For instance, I rediscovered a lot of the results that were known about multiply transitive groups, in particular, on the strictly 4-fold and 5-fold transitive groups. Fortunately, I did this with other methods than the ones that were used before. So these results were in fact new in a certain sense.

R & S: *Was it a disappointment to discover that these results had been discovered earlier?*

Tits: Not too much.

R & S: *Burnside was also interesting because he posed problems and conjectures that you and others worked on later, right?*

Thompson: Right—well, I sort of got started on working on the Frobenius conjecture, which was still open. I think it was Reinhold Baer or maybe Marshall Hall who told me about the Frobenius conjecture: The Frobenius kernel of the Frobenius group was conjectured to be nilpotent. I liked that conjecture for the following reason: If you take the group of proper motions of the Euclidean plane, it is a geometric fact that every proper motion is either a translation or is a rotation. I hope kids are still learning that. It is a curious phenomenon. And the translations form a normal subgroup. So that is something you could actually trace back to antiquity.

No doubt Frobenius knew that. So when he proved his theorem about the existence of the normal complement, that was a link back to very old things to be traced in geometry, I feel. That was one of the appeals. And then the attempt to use Sylow's theorems and a bit of character theory, whatever really, to deal with that problem. That is how I first got really gripped by pure mathematics.

R & S: *Mathieu discovered the first sporadic simple groups, the Mathieu groups, in the 1860s and 1870s. Why do you think we had to wait one hundred years before the next sporadic group was found by Janko, after your paper with Feit? Why did it take so long a time?*

Thompson: Part of the answer would be the flow of history. The attention of the mathematical community was drawn in other directions. I wouldn't say that group theory, certainly not finite group theory, was really at the center of mathematical development in the nineteenth century. For one thing, Riemann came along, topology gained and exerted tremendous influence, and as Jacques has mentioned, analysis was very deep and attracted highly gifted mathematicians. It is true, as you mentioned earlier, that Frobenius was there and Burnside; so group theory wasn't completely in the shadows. But there wasn't a lot going on.

Now, of course, there is a tremendous amount going on, both within pure and applied mathematics. There are many things that can attract people, really. So why there was this gap between these groups that Mathieu found and then the rather rapid development in the last half of the twentieth century of the simple groups, including the sporadic groups, I have to leave that to the historians. But I don't find it all that mysterious, really. You know, mathematics is a very big subject.

Jacques Tits receives the Abel Prize from King Harald. John Griggs Thompson to the left with the prize.

The Feit-Thompson Theorem

R & S: *The renowned Feit-Thompson theorem—finite groups of odd order are solvable—that you proved in the early 1960s: that was originally a conjecture by Burnside, right?*

Thompson: Burnside had something about it, yes. And he actually looked at some particular integers and proved that groups of that order were solvable. So he made a start.

R & S: *When you and Feit started on this project were there any particular results preceding your attack on the Burnside conjecture that made you optimistic about being able to prove it?*

Thompson: Sure. A wonderful result of Michio Suzuki, the so-called CA theorem. Absolutely basic! Suzuki came to adulthood just at the end of the Second World War. He was raised in Japan. Fortunately, he came to the University of Illinois. I think it was in 1952 that he published this paper on the CA groups of odd order and proved they were solvable by using exceptional character theory. It is not a very long paper. But it is incredibly ingenious, it seems to me. I still really like that paper. I asked him later how he came about it, and he said he thought about it for two years, working quite hard. He finally got it there. That was the opening wedge for Feit and me, really. The wedge got wider and wider.

Tits: Could you tell me what a CA group is?

Thompson: A CA group is a group in which the centralizer of every non-identity element is abelian. So we can see Abel coming in again: Abelian centralizer, that is what the A means.

R & S: *The proof that eventually was written down by Feit and you was 255 pages long, and it took one full issue of the* Pacific Journal *to publish.*

Thompson: It was long, yes.

R & S: *It is such a long proof and there were so many threads to connect. Were you nervous that there was a gap in this proof?*

Thompson: I guess so, right. It sort of unfolded in what seemed to us a fairly natural way; part group theory, part character theory, and this funny little number-theoretic thing at the end. It all seemed to fit together. But we could have made a mistake, really. It has been looked at by a few people since then. I don't lose sleep about it.

R & S: *It seems that, in particular in finite group theory, there did not exist that many connections to other fields of mathematics like analysis, at least at the time. This required that you had to develop tools more or less from scratch, using ingenious arguments. Is that one of the reasons why the proofs are so long?*

Thompson: That might be. It could also be that proofs can become shorter. I don't know whether that will be the case. I certainly can't see that the existing proofs will become tremendously shorter in my lifetime. These are delicate things that need to be explored.

Tits: You see, there are results that are intrinsically difficult. I would say that this is the case of the Feit-Thompson result. I personally don't believe that the proof will be reduced to scratch.

Thompson: I don't know whether it will or not. I don't think mathematics has reached the end of its tether, really.

Tits: It may of course happen that one can go around these very fine proofs, like John's proof, using big machinery like functional analysis. That one suddenly gets a big machine which crushes the result. That is not completely impossible. But the question is whether it is worth the investment.

The Theory of Buildings

R & S: *Professor Tits, you mentioned already Lie groups as a point of departure. Simple Lie groups had already been classified to a large extent at the end of the nineteenth century, first by Killing and then by Élie Cartan, giving rise to a series of matrix groups and the five exceptional simple Lie groups. For that purpose, the theory of Lie algebras had to be developed. When you started to work on linear algebraic groups, there were not many tools available. Chevalley had done some pioneering work, but the picture first became clear when you put it in the framework of buildings, this time associating geometric objects to groups. Could you explain to us how the idea of buildings, consisting of apartments, chambers, all of these suggestive words, how it was conceived, what it achieved, and why it has proven to be so fruitful?*

Tits: First of all, I should say that the terminology like buildings, apartments, and so on is not mine. I discovered these things, but it was Bourbaki who gave them these names. They wrote about my work and found that my terminology was a shambles. They put it in some order, and this is how the notions like apartments and so on arose.

I studied these objects because I wanted to understand these exceptional Lie groups geometrically. In fact, I came to mathematics through projective geometry; what I knew about was projective geometry. In projective geometry you have points, lines, and so on. When I started studying exceptional groups I sort of looked for objects of the same sort. For instance, I discovered—or somebody else discovered, actually—that the group E_6 is the collineation group of the octonion projective plane. And a little bit later, I found some automatic way of proving such results, starting from the group to reconstruct the projective plane. I could use this procedure to give geometric interpretations of the other exceptional groups, e.g., E_7 and E_8. That was really my starting point.

Then I tried to make an abstract construction of these geometries. In this construction I used terms like skeletons, for instance, and what became apartments were called skeletons at the time. In fact, in one of the volumes of Bourbaki, many of the exercises are based on my early work.

R & S: *An additional question about buildings: This concept has been so fruitful and made connections to many areas of mathematics that maybe you didn't think of at the time, like rigidity theory for instance?*

Tits: For me it was really the geometric interpretations of these mysterious groups, the exceptional groups, that triggered everything. Other people have then used these buildings for their own work. For instance, some analysts have used them. But in the beginning I didn't know about these applications.

R & S: *You asked some minutes ago about CA groups. Maybe we can ask you about BN-pairs: what are they and how do they come in when you construct buildings?*

Tits: Again, you see, I had an axiomatic approach towards these groups. The BN-pairs were an axiomatic way to prove some general theorems about simple algebraic groups. A BN-pair is a pair of two groups, B and N, with some simple properties. I noticed that these properties were sufficient to prove, I wouldn't say deep, but far-reaching results, for instance, proving the simplicity property. If you have a group with a BN-pair you have simple subgroups free of charge. The notion of BN-pairs arises naturally in the study of split simple Lie groups. Such groups have a distinguished conjugacy class of subgroups, namely the Borel subgroups. These are the Bs of a distinguished class of BN-pairs.

The Classification of Finite Simple Groups

R & S: *We want to ask you, Professor Thompson, about the classification project, the attempt to classify all finite simple groups. Again, the paper by Feit and you in 1962 developed some techniques. Is it fair to say that without that paper the project would not have been doable or even realistic?*

Thompson: That I can't say.

Tits: I would say yes.

Thompson: Maybe, but the history has bifurcations so we don't know what could have happened.

R & S: *The classification theorem for finite simple groups was probably the most monumental collaborative effort done in mathematics, and it was pursued over a long period of time. Many people have been involved, the final proof had 10,000 pages, at least, originally. A group of people, originally led by Gorenstein, are still working on making the proof more accessible.*

We had an interview here five years ago with the first Abel Prize recipient Jean-Pierre Serre. At that time, he told us that there had been a gap in the proof, that only was about to be filled in at the time of the interview with him. Before, it would have been premature to say that one actually had the proof. The quasi-thin case was left.

How is the situation today? Can we really trust that this theorem finally has been proved?

Thompson: At least that quasi-thin paper has been published now. It is quite a massive work itself, by Michael Aschbacher and Stephen Smith—quite long, well over 1,000 pages. Several of the sporadic simple groups come up. They characterize them because they are needed in quasi-thin groups. I forget which ones come up, but the Rudvalis group certainly is among them. It is excruciatingly detailed. It seems to me that they did an honest piece of work. Whether one can really believe these things is hard to say. It is such a long proof that there might be some basic mistakes. But I sort of see the sweep of it, really. It makes sense to me now. In some way it rounded itself off. I can sort of see why there are probably no more sporadic simple groups, but not really conceptually. There is no conceptual reason that is really satisfactory.

But that's the way the world seems to be put together. So we carry on. I hope people will look at these papers and see what the arguments are and see how they fit together. Gradually this massive piece of work will take its place in the accepted canon of mathematical theorems.

Tits: There are two types of group theorists. Those who are like St. Thomas: they don't believe because they have not seen every detail of the proof. I am not like them, and I believe in the final result although I don't know anything about it. The people who work on or who have worked on the classification theorem may of course have forgotten some little detail somewhere. But I don't believe these details will be very important. And I am pretty sure that the final result is correct.

R & S: *May we ask about the groups that are associated with your names? You have a group that's called the Thompson group among the sporadic simple groups. How did it pop up? How were you involved in finding it?*

Thompson: That is in fact a spin-off from the Monster Group. The so-called Thompson group is essentially the centralizer of an element of order three in the Monster. Conway and Norton and several others were beavering away—this was before Griess constructed the Monster—working on the internal structure where this group came up, along with the Harada-Norton group and the Baby Monster. We were all working trying to get the characters.

The Monster itself was too big. I don't think it can be done by hand. Livingstone got the character table, the ordinary complex irreducible characters of the Monster. But I think he made very heavy use of a computing machine. And I don't think that has been eliminated. That's how the figure 196,883 came about, the degree of the smallest faithful complex representation of the Monster Group. It is just too big to be done by hand. But we can do these smaller subgroups.

R & S: *The Tits group was found by hand, wasn't it? And what is it all about?*

Tits: Yes, it was really sort of a triviality. One expects that there would be a group there except that one must take a subgroup of index two so that it becomes simple. And that is what I know about this.

R & S: *Professor Tits, there is a startling connection between the Monster Group, the biggest of these sporadic groups, and elliptic function theory or elliptic curves via the j-function. Are there some connections with other exceptional groups, for instance in geometry?*

Tits: I am not a specialist regarding these connections between the Monster Group, for instance, and modular functions. I don't really know about these things, I am ashamed to say. I think it is not only the Monster that is related to modular forms, also several other sporadic groups. But the case of the Monster is especially satisfactory because the relations are very simple in that case. Somehow smaller groups give more complicated results. In the case of the Monster, things sort of round up perfectly.

The Inverse Galois Problem

R & S: *May we ask you, Professor Thompson, about your work on the inverse Galois problem? Can you explain first of all what the problem is all about? And what is the status right now?*

Thompson: The inverse Galois problem probably goes back already to Galois. He associated a group to an equation, particularly to equations in one variable with integer coefficients. He then associated to this equation a well-defined group now called the Galois group, which is a finite group. It captures quite a bit of the nature of the roots, the zeros, of this equation. Once one has the notion of a field, the field generated by the roots of an equation has certain automorphisms, and these automorphisms give us Galois groups.

The inverse problem is: Start with a given finite group. Is there always an equation, a polynomial with one indeterminate with integer coefficients, whose Galois group is that particular group? As far as I know it is completely open whether or not this is true. And as a test case, if you start with a given finite simple group, does it occur in this way? Is there an equation waiting for it? If there is one equation there would be infinitely many of them. So we wouldn't know how to choose a standard canonical equation associated to this group. Even in the case of simple groups, the inverse problem of Galois theory is not solved. For the most general finite groups, I leave it to the algebraic geometers or whoever else has good ideas whether this problem is amenable. A lot of us have worked on it and played around with it, but I think we have just been nibbling at the surface.

For example the Monster is a Galois group over the rationals. You can't say that about all sporadic groups. The reason that the Monster is a Galois group over the rationals comes from character theory. It is just given to you.

Tits: This is very surprising: you have this big object, and the experts can tell you that it is a Galois group. In fact, I would like to see an equation.

R & S: *Is there anything known about an equation?*

Thompson: It would have to be of degree of at least 1020. I found it impressive, when looking a little bit at the j-function literature before the days of computers, that people like Fricke and others could do these calculations. If you look at the coefficients of the j-functions, they grow very rapidly into the tens and hundreds of millions. They had been computed in Fricke's book. It is really pleasant to see these numbers out there before computers were around. Numbers of size 123 millions. And the numbers had to be done by hand, really. And they got it right.

Tits: It is really fantastic what they have done.

R & S: *Could there be results in these old papers by Fricke and others that people are not aware of?*

Thompson: No, people have gone through them, they have combed through them.

Tits: Specialists do study these papers.

The E_8 Story

R & S: *There is another collaborative effort that has been done recently, the so-called E_8 story: a group of mathematicians has worked out the representations of the E_8. In fact, they calculated the complete character table for E_8. The result was publicized last year in several American newspapers under the heading "A calculation the size of Manhattan" or something like that.*

Thompson: It was a little bit garbled maybe. I did see the article.

R & S: *Can you explain why we all should be interested in such a result? Be it as a group theorist, or as a general mathematician, or even as man on the street?*

Thompson: It is interesting in many ways. It may be that physicists have something to do with the newspapers. Physicists, they are absolutely fearless as a group. Any mathematical thing they can make use of they will gobble right up and put in a context that they can make use of, which is good. In that sense mathematics is a handmaiden for other things. And the physicists have definitely gotten interested in exceptional Lie groups. And E_8 is out there, really. It is one of the great things.

R & S: *Is there any reason to believe that some of these exceptional groups or sporadic groups tell us something very important—in mathematics or in nature?*

Thompson: I am not a physicist. But I know physicists are thinking about such things, really.

Tits: It is perhaps naive to say this, but I feel that mathematical structures that are so beautiful like the Monster must have something to do with nature.

Mathematical Work

R & S: *Are there any particular results that you are most proud of?*

Thompson: Well, of course one of the high points of my mathematical life was the long working relationship I had with Walter Feit. We enjoyed being together and enjoyed the work that we did and of course the fusion of ideas. I feel lucky to have had that contact and proud that I was in the game there.

Tits: I had a very fruitful contact for much of my career with François Bruhat, and it was very pleasant to work together. It was really working together like you did, I suppose, with Walter Feit.

R & S: *Was not Armand Borel also very important for your work?*

Tits: Yes, I also had much collaboration with Borel. But that was different in the following sense: when I worked with Borel, I had very often the impression that we both had found the same thing. We just put the results together in order not to duplicate. We wrote our papers practically on results that we had both found separately. Whereas with Bruhat, it was really joint work, complementary work.

R & S: *Have either of you had the lightning flash experience described by Poincaré, of seeing all of a sudden the solution to a problem you had struggled with for a long time?*

Tits: I think this happens pretty often in mathematical research, that one suddenly finds that something is working. But I cannot recall a specific instance. I know that it has happened to me and it has happened to John, certainly. So certainly some of the ideas one had work out, but it sort of disappears in a fog.

Thompson: I think my wife will vouch for the fact that when I wake in the morning I am ready to get out there and get moving right away. So my own naive thinking is that while I am asleep there are still things going on. And you wake up and say: "Let's get out there and do it." And that is a wonderful feeling.

R & S: *You have both worked as professors of mathematics in several countries. Could you comment on the different working environments at these places and people you worked with and had the best cooperation with?*

Tits: I think the country that has the best way of working with young people is Russia. Of course, the French have a long tradition, and they have very good, very young people. But I think Russian mathematics is in a sense more lively than French mathematics. French mathematics is too immediately precise. I would say that these are the two countries where the future of mathematics is the clearest. But of course Germany has had such a history of mathematics that they will continue. And nowadays, the United States have in a sense become the center of mathematics, because they have so much money. That they can…

R & S: *…buy the best researchers.*

Tits: That's too negative a way of putting it. Certainly many young people go the United States because they cannot earn enough money in their own country.

R & S: *And of course the catastrophe that happened in Europe in the 1930s with Nazism. A lot of people went to the United States.*

What about you, Professor Thompson? You were in England for a long time. How was that experience compared to work at an American university?

Thompson: Well, I am more or less used to holding my own role. People didn't harass me very much anyplace. I have very nice memories of all the places I have visited, mainly in the United States. But I have visited several other countries, too, for shorter periods, including Russia, Germany, and France. Mathematically, I feel pretty much comfortable everywhere I am. I just carry on. I have not really been involved in higher educational decision making. So in that sense I am not really qualified to judge what is going on at an international basis.

Thoughts on the Development of Mathematics

R & S: *You have lived in a period with a rapid development of mathematics, in particular in your own areas, including your own contributions. Some time ago, Lennart Carleson, who received the Abel Prize two years ago, said in an interview that the twentieth century had possibly been the Golden Age of Mathematics and that it would be difficult to imagine a development as rapid as we have witnessed.*

What do you think: Have we already had the Golden Age of Mathematics, or will development continue even faster?

Tits: I think it will continue at its natural speed, which is fast, faster than it used to be.

Thompson: I remember reading a quote attributed to Laplace. He said that mathematics might become so deep, that we have to dig down so deep, that we will not be able to get down there in the future. That's a rather scary image, really. It is true that prerequisites are substantial, but people are ingenious. Pedagogical techniques might change. Foundations of what people learn might alter. But mathematics is a dynamic thing. I hope it doesn't stop.

Tits: I am confident that it continues to grow.

R & S: *Traditionally, mathematics has been mainly linked to physics. Lots of motivations come from there, and many of the applications are towards physics. In recent years, biology, for example with the Human Genome Project, economics with its financial mathematics, and computer science and computing have been around, as well. How do you judge these new relations? Will they become as important as physics for mathematicians in the future?*

Tits: I would say that mathematics coming from physics is of high quality. Some of the best results we have in mathematics have been discovered by physicists. I am less sure about sociology and human science. I think biology is a very important subject but I don't know whether it has suggested very deep problems in mathematics. But perhaps I am wrong. For instance, I know of Gromov, who is a first class mathematician, and who is interested in biology now. I think that this is a case where really mathematics, highbrow mathematics, goes along with biology. What I said before about sociology, for instance, is not true for biology. Some biologists are also very good mathematicians.

Thompson: I accept that there are very clever people across the intellectual world. If they need mathematics they come up with mathematics. Either they tell mathematicians about it or they cook it up themselves.

Thoughts on the Teaching of Mathematics

R & S: *How should mathematics be taught to young people? How would you encourage young people to get interested in mathematics?*

Thompson: I always give a plug for Gamow's book *One Two Three … Infinity* and Courant and Robbins' *What is Mathematics?* and some of the expository work that you can get from the libraries. It is a wonderful thing to stimulate curiosity. If we had recipes, they would be out there by now. Some children are excited, and others are just not responsive, really. You have the same phenomenon in music. Some children are very responsive to music, others just don't respond. We don't know why.

Tits: I don't know what to say. I have had little contact with very young people. I have had very good students, but always advanced students. I am sure it must be fascinating to see how young people think about these things. But I have not had the experience.

R & S: *Jean-Pierre Serre once said in an interview that one should not encourage young people to do mathematics. Instead, one should discourage them. But the ones that, after this discouragement, still are eager to do mathematics, you should really take care of them.*

Thompson: That's a bit punitive. But I can see the point. You try to hold them back and if they strain at the leash then eventually you let them go. There is something to it. But I don't think Serre would actually lock up his library and not let the kids look at it.

R & S: *Maybe he wants to stress that research mathematics is not for everyone.*

Thompson: Could be, yeah.

Tits: But I would say that, though mathematics is for everyone, not everyone can do it with success. Certainly it is not good to encourage young people who have no gift to try to do something, because that will result in sort of a disaster.

Personal Interests

R & S: *In our final question we would like to ask you both about your private interests besides mathematics. What are you doing in your spare time? What else are you interested in?*

Tits: I am especially interested in music and, actually, also history. My wife is a historian; therefore I am always very interested in history.

R & S: *What type of music? Which composers?*

Tits: Oh, rather ancient composers.

R & S: *And in history, is that old or modern history?*

Tits: Certainly not contemporary history, but modern and medieval history. All related to my wife's speciality.

Thompson: I would mention some of the same interests. I like music. I still play the piano a bit. I like to read. I like biographies and history; general reading, both contemporary and older authors. My wife is a scholar. I am interested in her scholarly achievements. Nineteenth century Russian literature—this was a time of tremendous achievements. Very interesting things! I also follow the growth of my grandchildren.

Tits: I should also say that I am very interested in languages, Russian, for instance.

R & S: *Do you speak Russian?*

Tits: I don't speak Russian. But I have been able to read some Tolstoy in Russian. I have forgotten a little. I have read quite a lot. I have learned some Chinese. In the course of years I used to spend one hour every Sunday morning studying Chinese. But I started a little bit too old, so I forgot what I learned.

R & S: *Are there any particular authors you like?*

Tits: I would say all good authors.

Thompson: I guess we are both readers. Endless.

R & S: *Let us finally thank you very much for this pleasant interview, on behalf of the Norwegian, the Danish, and the European Mathematical Societies. Thank you very much.*

Thompson: Thank you.

Tits: Thank you for the interview. You gave us many interesting topics to talk about!